

The Quantum as an Emergent System

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Abstract

Double slit interference is explained with the aid of what we call “21st century classical physics”. We model a particle as an oscillator (“bouncer”) in a thermal context, which is given by some assumed “zero-point” field of the vacuum. In this way, the quantum is understood as an *emergent system*, i.e., a steady-state system maintained by a constant throughput of (vacuum) energy. To account for the particle’s thermal environment, we introduce a “path excitation field”, which derives from the thermodynamics of the zero-point vacuum and which represents all possible paths a particle can take via thermal path fluctuations. The intensity distribution on a screen behind a double slit is calculated, as well as the corresponding trajectories and the probability density current. Further, particular features of the relative phase are shown to be responsible for nonlocal effects not only in ordinary quantum theory, but also in our classical approach.

Keywords: quantum mechanics, ballistic diffusion, nonequilibrium thermodynamics, zero-point fluctuations

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1. INTRODUCTION: “NATURAL EXPLANATIONS” AND EMERGENCE

In his book review of Steve Adler’s *Quantum Theory as an Emergent Phenomenon* [1], Philip Pearle illustrated the concept of an “enlightenment task” by “...trying to explain the unnatural by the natural – in this case, the ‘unnatural’ being quantum physics and the ‘natural’ being classical physics. . .” [2]. Such tasks were already undertaken in the formative years of quantum theory, and Pearle characterized them very aptly in the following way:

“Classical particles and their dynamics are re-introduced, but a strong element of the unnatural remains. In the deBroglie-Bohm and Madelung models, it is the mysterious quantum force. In the Nelson model, it is the mysterious backward diffusion process (which, together with the usual classical forward diffusion process, forces a particle’s drift – its mean position – to be a dynamically determined quantity instead of, as classically, an independent variable set by external influences).”

Now, the “mysterious quantum force” ultimately derives from the “mysterious” wave function Ψ , a solution of the Schrödinger equation, which is an initial ingredient for any Bohmian-type model [3–5]. Whereas in the latter the existence of Ψ is usually not questioned and thus remains “unnatural” in Pearle’s terms, the Nelsonian model actually is an attempt to rely on Newtonian physics only [6–8], or on what one may thus also call “natural explanations”. Pearle’s comment apparently leaves two possibilities to overcome the “mysterious” element of the Nelson model, i.e., of backward diffusion. One would be to substitute the combination of forward and backward diffusion processes by some other process where a particle’s drift would turn out as a simple classical variable determined by external influences. As this is not viable since, upon the introduction of any usual forward diffusion, one does need a balancing process in opposition to it, the second possibility remains that a particle’s drift actually *is* a dynamically determined quantity, albeit in a new framework. This framework, we propose, would have to be some kind of steady-state maintained by a throughput of energy from the (“contextual”) environment.

In our “sub-quantum thermodynamics” approach to model quantum systems [9–15], we pursue the last option as a concrete possibility: not only do we consider *quantum theory* as emergent, but also, more specifically, the *quantum systems* themselves. This means that we refrain from any attempt to model the quantum on some singular, “basic” level only, but rather consider it as a “self-organizing system” whose description requires a more

encompassing combination of levels. One of the first definitions in science of such a system was given by Heinz von Foerster, i.e., as “a system which maps order of its environment onto its own organization.” [16] From the 1960ies onward, an ever increasing number of studies has been published, in a variety of different disciplines, on self-organization, and later on emergence, respectively. This tendency finally also entered the discussions on the foundations of physics, notably in the works by Ilya Prigogine [17], or in famous essays by P. W. Anderson (“More is Different” [18]), and later by Sam Schweber [19], as well as recently in popularized form by Robert B. Laughlin [20]. In a slight shift of emphasis from self-organization to emergence, Sam Schweber characterized the latter in the following way:

“It is not enough to know the ‘fundamental’ laws at a given level. It is the solutions to equations, not the equations themselves, that provide a mathematical description of the physical phenomena. ‘Emergence’ refers to properties of the solutions – in particular, the properties that are not readily apparent from the equations.” [19]

Schweber thus refers to the possibility that the “fundamental laws” on some “basic level” may by themselves not always be sufficient to grasp the essence of some physical phenomenon. This is reflected also in our group’s approach where the quantum is modeled as a self-organizing, *dynamical* entity whose complete description needs more than just one (presumably “basic”) level. As mentioned, we thus propose *that a quantum system be considered as a well-coordinated emergent system*. In doing so, we consider particle-like and wave-like phenomena as the result of both stochastic and regular dynamical processes. A prototype of such a system is well known from classical physics, viz., the “bouncing droplets” of Couder’s group [21–25], which in fact exhibit a whole series of phenomena reminiscent of quantum ones. Analogously, our group has in recent years attempted to model a quantum as a non-equilibrium steady-state maintained by a permanent throughput of energy. Specifically, we consider a “particle” as a “bouncer” whose oscillations are phase-locked with those of the energy-momentum reservoir of the surrounding “vacuum”, i.e., the zero-point field. (Note that the possible existence of a corresponding, underlying “medium” is *a priori* independent of quantum theory. For a similar view, compare, for example, the approach of “stochastic electrodynamics” by Cetto and de la Peña [26, 27]. For similar approaches in terms of assuming some sub-quantum, thermodynamic or hybrid-type variant of emergent quantum physics, see [1, 28–47])

In other words, we attempt to model the quantum in a “classical” framework. Note,

however, that we consider this framework more specifically to comprise what one may call “21st century classical physics”, i.e., including all the recent developments in the fields of nonequilibrium thermodynamics, ballistic diffusion, diffusion wave fields, and the like. More concretely, in assuming that (part of) the “classical” zero-point fluctuations undergo regular oscillatory motion, where the latter is partly caused by and dynamically coupled to the oscillator’s frequency ω , we are able to derive fundamental elements of quantum theory from a purely classical approach. In Ref. [14], as well as in Schwabl [48], we have shown how Planck’s relation between the energy E and the frequency ω , $E = \hbar\omega$, can be derived from a sub-quantum physics, with Planck’s (reduced) constant \hbar indicating a universal angular momentum, and we have also shown that with this relation alone one can derive the exact Schrödinger equation from (modern) classical physics [9, 10]. Moreover, also the stochastic element of the zero-point fluctuations enters decisively into our model, such that in effect we obtain an exact description of free quantum motion via a combination of the propagation of classical Huygens-type waves with diffusion due to stochastic sub-quantum mechanics [11]. We particularly stress that the “particle” is considered as an off-equilibrium steady state oscillator maintained by a constant throughput of energy provided by the zero-point field. Thus, it is exactly this *energy throughput* which is responsible for a particle’s *natural drift* in Pearle’s sense: it is the permanent absorption and re-emission of kinetic energy which will be explicated below to provide a “natural” diffusion model, albeit one of a specific kind, which is called “ballistic diffusion”. So, a quantum in our model emerges from the synchronized dynamical coupling between an oscillator (“bouncer”) and its wave-like environment. In sum, with this ansatz, we have been able to derive from “21st century classical physics” the following quantum mechanical features:

- Planck’s relation for the energy of a particle,
- the exact (one- and n-particle) Schrödinger equation for conservative and non-conservative systems,
- the Heisenberg uncertainty relations,
- the quantum mechanical superposition principle and Born’s rule,
- the quantum mechanical decay of a Gaussian wave packet,

- quantum mechanical interference at the double slit.

Although in our model Huygens’ principle applies, it does so only for an idealized combination of a “walking” motion of some velocity \mathbf{v} with the centrally-symmetric diffusion waves’ motions of velocity \mathbf{u} orthogonal to it. Effectively, it only holds approximately, disturbed by that part of the accompanying diffusive process which is to be described by the corresponding velocity fluctuations $\delta\mathbf{u}$. In fact, as we have shown for a Gaussian slit, the exact quantum mechanical result can be described as a combination of classical wave mechanics with the addition of a corresponding stochastic diffusion process [11]. We have also shown [15] that the same modelling procedure also perfectly applies to a double slit system. In particular, to make our point as clear as possible, we provide here a more extensive discussion of a simple calculational tool related to what we call the “path excitation field”. With it, one can easily derive results for quantum mechanics without ever using complex-valued functions such as wave functions, for example.

2. THE PATH EXCITATION FIELD: A CLASSICAL EXPLANATORY FRAMEWORK FOR GAUSSIAN DISPERSION AND DOUBLE-SLIT INTERFERENCE

To begin with, we recall some of the basic results of our earlier work, including that on diffraction at a single Gaussian slit [11]. We claim for a particle of frequency ω embedded in a stochastic (“zero-point”) environment that its *average total energy* is given by the average “total” energy $\overline{\hbar\omega}$ of the particle itself plus a kinetic energy term due to momentum changes $\mathbf{p}_u =: m\mathbf{u}$ which it receives from or gives off to the environment:

$$\overline{E_{\text{tot}}} = \overline{\hbar\omega} + \frac{\overline{p_u^2}}{2m} = \text{const}, \quad (2.1)$$

where the averaging (as denoted by the bars) is defined in n -dimensional configuration space as

$$\overline{p_u^2} := \int P p_u^2 d^n x. \quad (2.2)$$

$P = P(\mathbf{x}, t)$ refers to the probability density of some relevant distribution. For our model system, the latter is given as a solution of a generalized (“anomalous”) diffusion equation, i.e., with a time-dependent diffusion coefficient. As can be seen from (2.1), the momentum changes can be either positive or negative, and actually will *on the average* balance each

other, since they are *a priori* unbiased. The deeper reason for this balancing in our model is due to the fact that we consider the quantum to be a steady-state system in the sense that its “total” energy $\hbar\omega$ is maintained over times $t \gg 1/\omega$ by the permanent throughput of kinetic energy $p_u^2/2m$. In other words, to maintain the steady-state, during the intervals of the average order of $t \simeq 1/\omega$, there will both be an absorption of a momentum $p_u = mu$ and a release of the same amount, $p_u = -mu$, thus providing a “natural” explanation of the involved diffusion processes as envisaged in the introduction.

In our earlier papers [9–11], we have shown that, apart from the ordinary particle current $\mathbf{J}(\mathbf{x}, t) = P(\mathbf{x}, t)\mathbf{v}$, we are thus dealing with two additional, yet opposing, currents $\mathbf{J}_u = P(\mathbf{x}, t)\mathbf{u}$, which are on average orthogonal to \mathbf{J} [9–11, 14], and which are the emergent outcome from the presence of numerous corresponding velocities

$$\mathbf{u}_{\pm} = \mp \frac{\hbar}{2m} \frac{\nabla P}{P}. \quad (2.3)$$

We denote with \mathbf{u}_+ and \mathbf{u}_- , respectively, the two opposing tendencies of the diffusion process. In the reference frame of a single free particle, and starting at $t = 0$ at the center of the distribution P , the averages obey

$$\bar{\mathbf{u}}_-(\mathbf{x}, t) = -\bar{\mathbf{u}}_+(-\mathbf{x}, t). \quad (2.4)$$

Now let us consider an experimental setup with a particle source. To describe the velocity distribution, we introduce a velocity field with average velocity $\bar{\mathbf{v}}$, and amplitudes $R(\mathbf{x}, t)$. As mentioned, we refer to their intensities $P = R^2$ as the solutions of a diffusion equation. These typically appear in the form of a Gaussian distribution $P(\mathbf{x}, t)$ of possible particle locations \mathbf{x} , even if there is only one particle at a time emerging from the corresponding “Gaussian slit”, in one dimension for simplicity,

$$P(x, t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}}, \quad (2.5)$$

with the usual variance $\sigma^2 = \overline{(\Delta x)^2} = \overline{(x - x_0)^2}$, and where we choose $x_0 = 0$ further on. Regarding \mathbf{u} , even in this scenario of one-particle-at-a-time, we deal with an *ensemble* of velocity vectors $\mathbf{u}_{\alpha}(t)$ representing hypothetical motions on the sub-quantum level in a small volume around \mathbf{x} , whose mean value will be given by

$$\mathbf{u}(\mathbf{x}, t) = \frac{1}{N(\mathbf{x}, t)} \sum_{\alpha=1}^{N(\mathbf{x}, t)} \mathbf{u}_{\alpha}(t). \quad (2.6)$$

Here, the (typically very large) number N refers to the number of possible path directions of the bouncer due to the existence of the wave-like excitations of the zero-point field. This may be reminiscent of Feynman’s picture of photons *virtually* probing every possible path in an experimental setup, but in our case it is a configuration of *real* wave-like excitations, which in a resulting Brownian-type motion guide the bouncer along its path of average velocity \mathbf{u} . Again, we note that here we discuss not only a passive guidance of the “particle” by the surrounding wave configurations, but point out also the very active role of the “particle” in (partly) *creating* said wave configurations due to the effects of its bouncing. To account for (2.3), we split up $\mathbf{u}(\mathbf{x}, t)$ according to

$$\mathbf{u}(\mathbf{x}, t) = \frac{1}{2N} \left[\sum_{\alpha=1}^N \mathbf{u}_{\alpha,+} + \sum_{\alpha=1}^N \mathbf{u}_{\alpha,-} \right] = \frac{1}{2} [\mathbf{u}_+ + \mathbf{u}_-], \quad (2.7)$$

thus reflecting the isotropy of the diffusion process. Still, the uncontrollable and possibly unknowable velocity field \mathbf{u} representing the Brownian motion of the bouncer may not be operational, but when we take the average according to the rule (2.2), we obtain a “smoothed-out” *average velocity field*

$$\overline{\mathbf{u}(\mathbf{x}, t)} = \int P \mathbf{u}(\mathbf{x}, t) d^n x, \quad (2.8)$$

which is all that we need for our further considerations. Similarly, based on the fact that we have an initial Gaussian distribution of velocity vectors $\mathbf{v}(\mathbf{x}, t)$, we define an average velocity field $\overline{\mathbf{v}}$ of the wave propagation as

$$\overline{\mathbf{v}(\mathbf{x}, t)} = \int P \mathbf{v}(\mathbf{x}, t) d^n x, \quad (2.9)$$

and make use of an average orthogonality between the two velocity fields, \mathbf{u} and \mathbf{v} , [9–11, 13],

$$\overline{\mathbf{v} \cdot \mathbf{u}} = \int P \mathbf{v} \cdot \mathbf{u} d^n x = 0. \quad (2.10)$$

In effect, then, the combined presence of both velocity fields \mathbf{u} and \mathbf{v} can be denoted as a *path excitation field*: via diffusion, the bouncer in its interaction with already existing wave-like excitations of the environment creates an “agitated”, or “heated-up”, thermal “landscape”, which can also be pictured by interacting wave configurations all along between source and detector of an experimental setup. Recall that our prototype of a “walking bouncer”, i.e., from the experiments of Couder’s group, is always driven by its interactions

with a superposition of waves emitted at the points it visited in the past. Couder et al. denote this superposition of in-phase waves the “path memory” of the bouncer [25]. This implies, however, that the bouncers at the points visited in “the present” necessarily create new wave configurations which will form the basis of a “path memory” in the future. In other words, the wave configurations of the past determine the bouncer’s path in the present, whereas its bounces in the present co-determine the wave configurations at any of the possible locations it will visit in the future. Therefore, we call the latter configurations the *path excitation field*, which may also be described as “heated-up” thermal field. As in the coupling of an oscillator with classical diffusion, diffusion wave fields arise with instantaneous field propagation [10, 49], one has elements of the whole setup which may be nonlocally oscillating (“breathing”) in phase. This means that the Gaussian of (2.5) does represent a nonlocal path excitation field in that it is a physically existing and effective entity responsible for where the bouncing “particle” can possibly go. As we have shown [11], one can in this classical framework, along with the time-dependence of the diffusivity, effectively and easily describe the (sub-)quantum physics of diffraction at a Gaussian slit, which we now briefly recapitulate.

At first we note that Eq. (2.1) is an *average* energy conservation law only. This means that apart from the momentum changes $\mathbf{p}_u = \pm m\mathbf{u}$ discussed so far, also variations in \mathbf{p}_u will have to be taken into account, and thus also variations in the “particle energy” $\hbar\omega$. If for the latter one just considers its kinetic energy term, $mv^2/2$, then said variations will lead to exchanges of velocity/momentum terms providing the net balance

$$m\delta\mathbf{v} = m\delta\mathbf{u}. \quad (2.11)$$

Using the expression (2.3) for \mathbf{u} , one obtains with the Gaussian (2.5), $\overline{\mathbf{p}_u^2}|_{t=0} = m^2 u_0^2$, and with $\overline{(\nabla \ln P)^2} = -\overline{\nabla^2 \ln P}$, that

$$u_0^2 = \frac{D^2}{\sigma_0^2} = \overline{u^2} + \overline{(\delta u)^2} = \frac{D^2}{\sigma^2} + \overline{(\delta u)^2}, \quad (2.12)$$

where as usual $\sigma = \sigma(t) = \sqrt{x^2}$ for $x_0(t=0) = 0$, and $\sigma_0 = \sigma(t=0)$. One can view the Gaussian distribution P of kinetic energy also as a sort of “heat accumulation”, which has its maximum at the center. Considering now the application of momentum fluctuations (up to second order) to a particle with initial ($t=0$) distance $x(0)$ from said center, with the fluctuation term for $t > 0$ defined as $\mathbf{p}_u \pm \delta\mathbf{p}_u = \pm m(\mathbf{u} \pm \delta\mathbf{u})$, one obtains at time t the

envisaged “natural” drift as

$$\mathbf{x}(t) = \mathbf{x}(0) \pm (\mathbf{u} \pm \delta \mathbf{u}) t. \quad (2.13)$$

Squaring Eq. (2.13) and forming the r.m.s. can then easily be shown to provide [11],

$$\overline{x^2}(t) = \overline{x^2}(0) + \left[\overline{u^2} + \overline{(\delta u)^2} \right] t^2 = \overline{x^2}(0) + u_0^2 t^2. \quad (2.14)$$

Comparing with Eq. (2.12) also provides the time evolution of the wave packet’s variance as

$$\sigma^2 = \sigma_0^2 \left(1 + \frac{D^2 t^2}{\sigma_0^4} \right), \quad (2.15)$$

and finally the average velocity field of a Gaussian wave packet as

$$v_{\text{tot}}(t) = v(t) + [x(t)] \frac{u_0^2 t}{\sigma^2}. \quad (2.16)$$

Note that Eqs. (2.15) and (2.16) are derived solely from classical physics. Still, they are in full accordance with quantum theory, and in particular with Bohmian trajectories [5]. Note also that one can rewrite Eq. (2.14) such that it appears like a linear-in-time formula for Brownian motion,

$$\overline{x^2} = \overline{x^2(0)} + D(t) t, \quad (2.17)$$

where a time dependent diffusivity

$$D(t) = u_0^2 t = \frac{\hbar^2}{4m^2 \sigma_0^2} t \quad (2.18)$$

characterizes Eq. (2.17) as *ballistic diffusion*. The appearance of a time-dependent $D(t)$ is essential, but also straightforward. The diffusivity is changed over time, because the “particle’s” thermal environment changes: With the “heat” initially concentrated within the narrow spatial constraints determined by σ_0 of the source (“Gaussian slit”), $D(t)$ must become larger with time because of the gradually lower heat concentration due to dissipation into the unconstrained environment. (Note that a similar scenario was suggested by Garbaczewski [30]; others are presently intensively discussed in the context of the so-called “superstatistics” [50].) This makes it possible to simulate the dispersion of a Gaussian wave packet on a computer by simply employing coupled map lattices for classical diffusion, with the diffusivity given by Eq. (2.18). (For detailed discussions, see refs. [11] and [13].)

With the essentials of Gaussian dispersion at our disposal, it is very simple to now also describe and explain quantum interference with our approach. [15] We have chosen a

textbook scenario in the form of the calculation of the intensity distribution and the particle trajectories in an electron interferometer. As we are also interested in the trajectories, we refer to, and compare our results with, the well-known work by Phillipidis et al. [51], albeit in the form as presented by Holland [5].

We choose similar initial situations as in [5], i.e., electrons (represented by plane waves in the forward y -direction) from a source passing through “soft-edged” slits 1 and 2 in a barrier (located along the x -axis) and recorded at a screen. In our model, we therefore note two Gaussians representing the totality of the effectively “heated-up” path excitation field, one for slit 1 and one for slit 2, whose centers have the distances $+X$ and $-X$ from the plane spanned by the source and the center of the barrier along the y -axis, respectively.

As it is well known from classical wave mechanics, the total amplitude R of two coherent waves with (suitably normalized) amplitudes $R_i = \sqrt{P_i}$, $i = 1$ or 2 , is given by

$$\begin{aligned} R(\mathbf{k}_1, \mathbf{k}_2, \mathbf{r}) &= R_1 \cos(\omega t - \mathbf{k}_1 \cdot \mathbf{r} + \varphi_0) + R_2 \cos(\omega t - \mathbf{k}_2 \cdot \mathbf{r} + \varphi_0) \\ &= R_1 \cos(\omega t - \varphi_1) + R_2 \cos(\omega t - \varphi_2), \end{aligned} \quad (2.19)$$

where φ_0 is some initial phase at $t = 0$, and $\varphi_i = \mathbf{k}_i \cdot \mathbf{r} - \varphi_0$. Considering the average of this expression over a multitude of similar wave superpositions, one obtains as usual the *averaged total intensity*

$$P_{\text{tot}} := R^2 = R_1^2 + R_2^2 + 2R_1 R_2 \cos \varphi = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \varphi, \quad (2.20)$$

where $\varphi = \varphi_1 - \varphi_2 = (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r}$. Note that the relative phase difference φ enters Eq. (2.20) only via the cosine function, such that, e.g., even if the total wave numbers (and thus also the total momenta) \mathbf{k}_i were of vastly different size, the cosine effectively makes Eq. (2.20) independent of said sizes, but dependent only on an angle modulo 2π . This will turn out as essential for our discussion further below.

Now, the x -components of the centroids’ motions from the two alternative slits 1 and 2, respectively, are given by the “particle” velocity components

$$v_x = \pm \frac{\hbar}{m} k_x, \quad (2.21)$$

respectively, such that the relative group velocity of the Gaussians spreading into each other is given by $\Delta v_x = 2v_x$. However, in order to calculate the phase difference φ descriptive of the interference term of the intensity distribution (2.20), one must take into account the

total momenta involved, i.e, one must also include the wave packet dispersion as described in the previous Chapter. Thus, one obtains with the displacement $\pm x(t) = \mp (X + v_x t)$ in Eq. (2.16) the total relative velocity of the two Gaussians as

$$\Delta v_{\text{tot},x} = 2 \left[v_x - (X + v_x t) \frac{u_0^2 t}{\sigma^2} \right]. \quad (2.22)$$

Therefore, the total phase difference between the two possible paths (i.e., through either slit) becomes

$$\varphi = \frac{1}{\hbar} (m \Delta v_{\text{tot},x} x) = 2m v_x \frac{x}{\hbar} - (X + v_x t) x \frac{1}{D} \frac{u_0^2 t}{\sigma^2}. \quad (2.23)$$

The Gaussians P_1 and P_2 for the corresponding slits are given as

$$P_1(x, t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-[x-(X+v_x t)]^2/2\sigma^2}, \quad (2.24)$$

and

$$P_2(x, t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-[x+(X+v_x t)]^2/2\sigma^2}. \quad (2.25)$$

With equal amplitudes $R = R_i = \sqrt{P_i}$, for $i = 1, 2$, of the Gaussians, and with normalization constant N , we thus obtain the usual interference pattern in the form of the intensity distribution:

$$P_{\text{tot}}(x, t) = R^2 N^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-[x^2+(X+v_x t)^2]/2\sigma^2} \left\{ e^{x(X+v_x t)/\sigma^2} + e^{-x(X+v_x t)/\sigma^2} + 2 \cos \varphi \right\}, \quad (2.26)$$

with the relative phase φ given by Eq. (2.23). Whereas the first exponential describes the enveloping Gaussian, the term inside the curly brackets of Eq. (2.26) describes the interference fringes whose “dark” nodes are at the locations well-known from textbooks,

$$x = (n + \frac{1}{2})\pi/k_x, \quad (2.27)$$

if $X = -v_x t$, as the wave packets must approach each other ($v_x < 0$) for $t > 0$.

Fig. 2.1 depicts the interference of two beams emerging from Gaussian slits established with the aid of a purely classical simulation. As in the case of a Gaussian slit [11], we again simulate diffusion with a time-dependent diffusivity $D(t)$. To account for interference, we simply follow the classical rule for the intensities (2.20), with φ from Eq. (2.23). The averaged trajectories are the flux lines obtained by choosing a set of equidistant initial points at $y = 0$. Two adjacent flux lines thereby define regions of constant flux, i.e., $\int_A P \, dA = \text{const.}$, with A being the cross section of a flux tube.

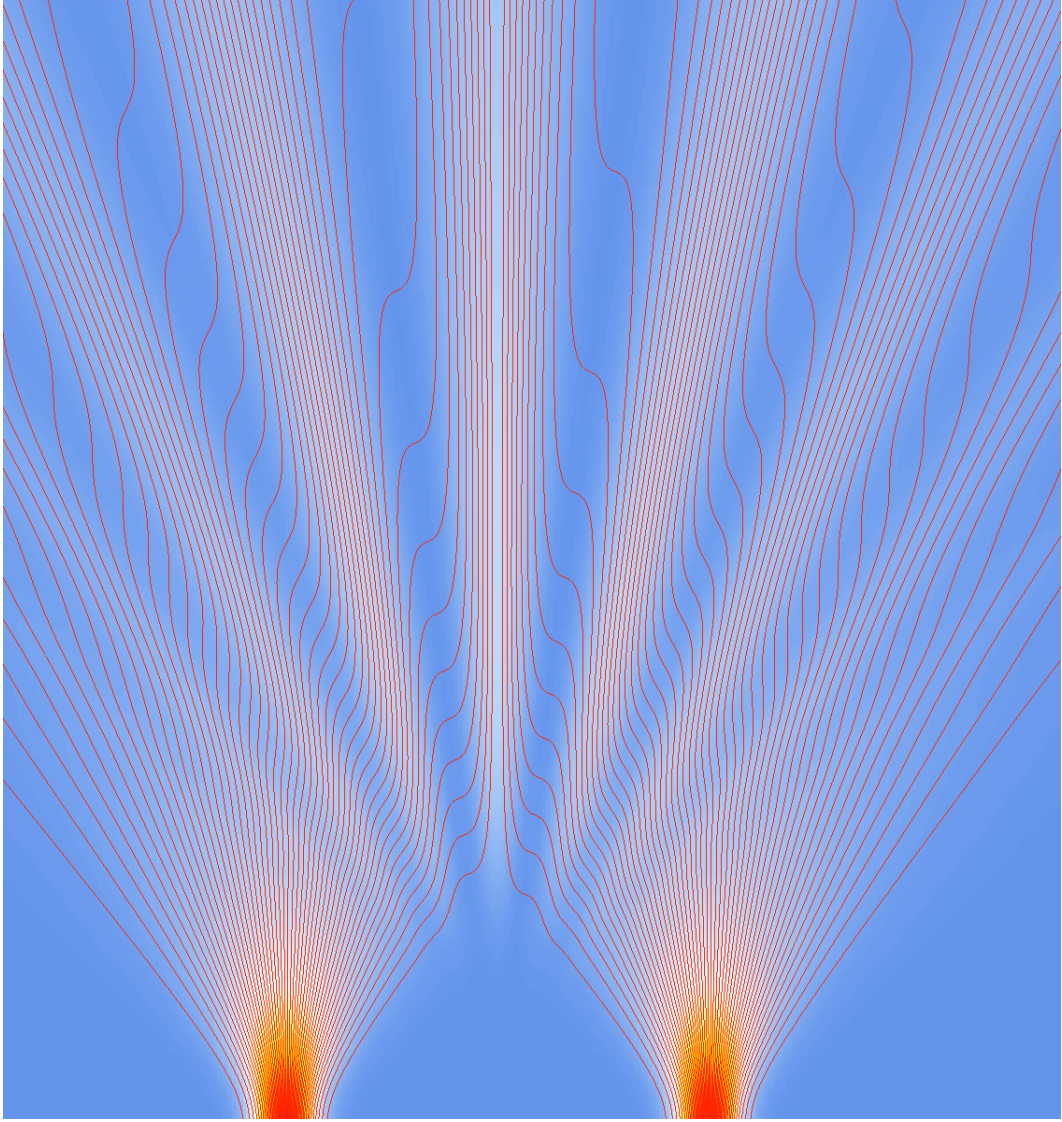


Figure 2.1. Classical computer simulation of the interference pattern: intensity distribution with increasing intensity from white through yellow and orange, with averaged trajectories (red) for two Gaussian slits, and with large dispersion (evolution from bottom to top; $\overline{v_{x,1}} = \overline{v_{x,2}} = 0$). The interference hyperbolas for the maxima characterize the regions where the phase difference $\varphi = 2n\pi$, and those with the minima lie at $\varphi = (2n + 1)\pi$, $n = 0, 1, 2, \dots$. Note in particular the “kinks” of trajectories moving from the center-oriented side of one relative maximum to cross over to join more central (relative) maxima. In our classical explanation of interference, a detailed micro-causal account of the corresponding kinematics can be given. The averaged trajectories follow a Bohm-type “no crossing” rule: particles from the left slit stay on the left side and *vice versa* for the right slit. This feature is explained here by a sub-quantum build-up of kinetic (heat) energy acting as an emergent repeller along the symmetry line.

Just as with the dispersion of a single Gaussian, we want to stress that also the quantum interference pattern (2.26) has been derived here without the use of quantum mechanics, but solely on the basis of classical physics. Exploiting our concept of the “path excitation field”, we have for this derivation implicitly used the assumption of velocity fields $\mathbf{u}(\mathbf{x}, t)$ and $\mathbf{v}(\mathbf{x}, t)$, respectively, which have entered the expression of a Gaussian’s average velocity field, Eq. (2.16), and which in turn were shown to essentially contribute to the interference pattern (2.26). In what follows, we now want to make the use of these velocity fields more explicit, i.e., we shall now concentrate on understanding the emerging particle trajectories during quantum interference on the basis of our classical velocity fields. As it turns out, a thorough consideration of the nature of the relative phase φ will make it possible to obtain a deeper understanding of quantum interference in general.

3. GEOMETRIC MEANING OF THE PATH EXCITATION FIELD

Let us consider a single, classical “particle” (“bouncer”) following the propagation of a set of waves of equal amplitude R_i , each representing one of i possible alternatives according to our principle of path excitation. We first note that for the superposition of two weighted wave vectors, with resultant vector \mathbf{k} and total amplitude R_{tot} ,

$$R_{\text{tot}}\mathbf{k} = R_1\mathbf{k}_1 + R_2\mathbf{k}_2, \quad (3.1)$$

where the *averaged scalar product* of the two associated unit vectors is given by $\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2 = \cos\varphi$ in accordance with (2.20).

We now focus on the specific role of the velocity fields \mathbf{u} , which were present in Section 2 only implicitly in the expressions (2.20) and (2.23). To describe the required details, each path i be occupied by a Gaussian wave packet with a “forward” momentum $\mathbf{p}_i = \hbar\mathbf{k}_i = m\mathbf{v}_i$. Moreover, due to the stochastic process of path excitation, the latter has to be represented also by a large number N of consecutive Brownian shifts, $\mathbf{p}_{u,\alpha} = m\mathbf{u}_\alpha$. Recalling (2.7), one obtains for the case of interference at a double slit the total averaged velocity field (with indices $i = 1$ or 2 referring to the two slits)

$$\bar{\mathbf{v}}_{\text{tot}} = \bar{\mathbf{v}}_{\text{tot},1} + \bar{\mathbf{v}}_{\text{tot},2} := \bar{\mathbf{v}}_1 + \frac{\bar{\mathbf{u}}_{1+}}{2} + \frac{\bar{\mathbf{u}}_{1-}}{2} + \bar{\mathbf{v}}_2 + \frac{\bar{\mathbf{u}}_{2+}}{2} + \frac{\bar{\mathbf{u}}_{2-}}{2}. \quad (3.2)$$

With two Gaussian distributions $P_1 = R_1^2$ and $P_2 = R_2^2$ as given in the previous Chapter,

one has the corollary of (3.1), i.e.,

$$R_{\text{tot}} \bar{\mathbf{v}}_{\text{tot}} = R_1 \bar{\mathbf{v}}_{\text{tot},1} + R_2 \bar{\mathbf{v}}_{\text{tot},2} , \quad (3.3)$$

and thus

$$R_{\text{tot}} = \left[R_1 \left(\bar{\mathbf{v}}_1 + \frac{\bar{\mathbf{u}}_{1+}}{2} + \frac{\bar{\mathbf{u}}_{1-}}{2} \right) + R_2 \left(\bar{\mathbf{v}}_2 + \frac{\bar{\mathbf{u}}_{2+}}{2} + \frac{\bar{\mathbf{u}}_{2-}}{2} \right) \right] \frac{\hat{\mathbf{v}}_{\text{tot}}}{|\bar{\mathbf{v}}_{\text{tot}}|}. \quad (3.4)$$

To help with the bookkeeping, the schematic of Fig. 3.1 displays all the relevant vectors and some of the corresponding phase angles. Taking into account the conservation of the “particle momentum” in both channels, we have $|\bar{\mathbf{v}}_{\text{tot}}| = |\bar{\mathbf{v}}_{\text{tot},1}| = |\bar{\mathbf{v}}_{\text{tot},2}|$, leading to

$$R_{\text{tot}} = \left[R_1 \left(\hat{\mathbf{v}}_1 + \frac{\hat{\mathbf{u}}_{1+}}{2} + \frac{\hat{\mathbf{u}}_{1-}}{2} \right) + R_2 \left(\hat{\mathbf{v}}_2 + \frac{\hat{\mathbf{u}}_{2+}}{2} + \frac{\hat{\mathbf{u}}_{2-}}{2} \right) \right] \hat{\mathbf{v}}_{\text{tot}}. \quad (3.5)$$

With Eq. (2.4), and setting $\mathbf{u}_{1+} \rightarrow \mathbf{u}_1$ and $\mathbf{u}_{1-} \rightarrow -\mathbf{u}_1$, one obtains the total average current

$$\begin{aligned} \bar{\mathbf{J}}_{\text{tot}} &= R_{\text{tot}}^2 \bar{\mathbf{v}}_{\text{tot}} = R_1^2 \bar{\mathbf{v}}_1 + R_2^2 \bar{\mathbf{v}}_2 \\ &+ R_1 R_2 \left\{ (\bar{\mathbf{v}}_1 + \bar{\mathbf{v}}_2) \cos(\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2) + \left(\bar{\mathbf{v}}_1 + \frac{\bar{\mathbf{u}}_2}{2} \right) \cos(\hat{\mathbf{v}}_1, \hat{\mathbf{u}}_2) - \left(\bar{\mathbf{v}}_1 - \frac{\bar{\mathbf{u}}_2}{2} \right) \cos(\hat{\mathbf{v}}_1, \hat{\mathbf{u}}_2) \right. \\ &+ \left(\frac{\bar{\mathbf{u}}_1}{2} + \bar{\mathbf{v}}_2 \right) \cos(\hat{\mathbf{u}}_1, \hat{\mathbf{v}}_2) - \left(-\frac{\bar{\mathbf{u}}_1}{2} + \bar{\mathbf{v}}_2 \right) \cos(\hat{\mathbf{u}}_1, \hat{\mathbf{v}}_2) + \left(\frac{\bar{\mathbf{u}}_1}{2} + \frac{\bar{\mathbf{u}}_2}{2} \right) \cos(\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2) \\ &\left. - \left(\frac{\bar{\mathbf{u}}_1}{2} - \frac{\bar{\mathbf{u}}_2}{2} \right) \cos(\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2) - \left(-\frac{\bar{\mathbf{u}}_1}{2} + \frac{\bar{\mathbf{u}}_2}{2} \right) \cos(\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2) + \left(-\frac{\bar{\mathbf{u}}_1}{2} - \frac{\bar{\mathbf{u}}_2}{2} \right) \cos(\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2) \right\}, \end{aligned} \quad (3.6)$$

and thus finally

$$\bar{\mathbf{J}}_{\text{tot}} = P_1 \bar{\mathbf{v}}_1 + P_2 \bar{\mathbf{v}}_2 + \sqrt{P_1 P_2} (\bar{\mathbf{v}}_1 + \bar{\mathbf{v}}_2) \cos \varphi + \sqrt{P_1 P_2} (\bar{\mathbf{u}}_1 - \bar{\mathbf{u}}_2) \sin \varphi. \quad (3.7)$$

Note that Eq. (3.7), upon the identification of $\bar{\mathbf{u}}_i = -\frac{\hbar}{m} \frac{\nabla R_i}{R_i}$ from Eq. (2.3) and with $P_i = R_i^2$, turns out to be in perfect agreement with a comparable “Bohmian” derivation [5, 52]. The formula for the averaged particle trajectories, then, simply results from Eq. (3.6), i.e.,

$$\bar{\mathbf{v}}_{\text{tot}} = \frac{\bar{\mathbf{J}}_{\text{tot}}}{P_{\text{tot}}} = \frac{P_1 \bar{\mathbf{v}}_1 + P_2 \bar{\mathbf{v}}_2 + \sqrt{P_1 P_2} (\bar{\mathbf{v}}_1 + \bar{\mathbf{v}}_2) \cos \varphi + \sqrt{P_1 P_2} (\bar{\mathbf{u}}_1 - \bar{\mathbf{u}}_2) \sin \varphi}{P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \varphi}. \quad (3.8)$$

In Fig. 2.1 one can observe a basic characteristic of the (averaged) particle trajectories, which, just because of the averaging, are identical with the Bohmian trajectories. In particular, due to the “no crossing” rule for Bohmian trajectories, the particles coming from, say, the right slit (and expected at the left part of the screen if a presumed “classical” momentum conservation should hold) actually arrive at the right part of the screen (and *vice versa* for the other slit). In our sub-quantum approach an explanation of the “no crossing” rule is

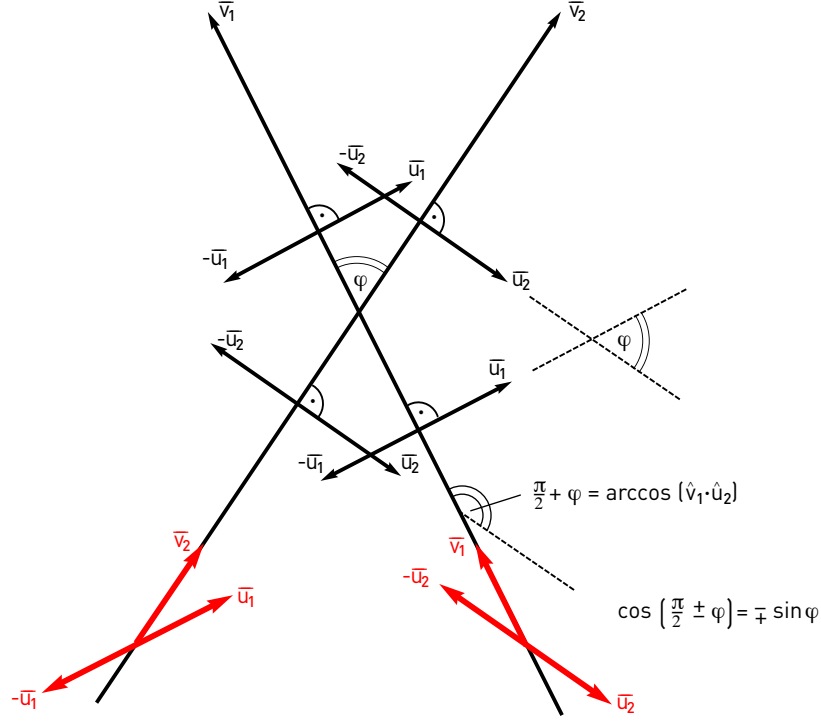


Figure 3.1. Schematic of the phase angles between various components of the path excitation field behind a double slit. \bar{u}_i and \bar{v}_i , $i = 1$ or 2 , denote the average velocity fields involved, with \bar{v}_i referring to single free “particle” velocities (i.e., either through slit 1 or slit 2), and the \bar{u}_i referring to additional diffusion velocities. The thick arrows (red online) indicate how the mixing of velocity fields \bar{v}_i and \bar{u}_j , with $i \neq j$, produces the terms proportional to $\sin \varphi$ in Eq. (3.7). Note that angles such as φ in the schematic symbolize the actual phase angles between any two vectors, and must therefore be understood as emerging out of the interaction of all the velocity/momentum vectors involved in a particular point in space. So, the schematic depicts the totality of all the “excited” velocity/momentum fields involved, labeled separately for each channel. Upon actual superposition of the two channels in x – space, i.e., in the course of the interactions among all these field excitations, one then obtains the final intensity distributions and, according to Eq. (3.8), the particle trajectories whose emergent behaviour turns out to be characterized by a “no crossing” rule (see Fig. 2.1). In other words, there is no simple linear superposition between the corresponding processes for each separate channel 1 and 2, respectively, but a complex evolution of all the involved sub-quantum processes that leads to the emergent interference pattern and the associated trajectories.

actually a consequence of a detailed microscopic momentum conservation. As can be seen in Fig. 2.1, the (Bohmian) trajectories are repelled from the central symmetry line. However, in our case this is only implicitly due to a “quantum potential”, but actually due to the identification of the latter with a kinetic (rather than a potential) energy: As has already been stressed in [10], it is the “heat of the compressed vacuum” that accumulates along said symmetry line (i.e., as reservoir of “outward” oriented kinetic energy) and therefore repels the trajectories.

The trajectories in Fig. 2.1 exactly obey Eq. (3.8) for the two Gaussian slits shown. The interference hyperbolas for the maxima characterize the regions where the phase difference $\varphi = 2n\pi$, and those with the minima lie at $\varphi = (2n + 1)\pi$, $n = 0, 1, 2, \dots$. Note in particular the “kinks” of trajectories moving from the center-oriented side of one relative maximum to cross over to join more central (relative) maxima. In addition to the full accordance with the trajectories obtained from the Bohmian approach (see [4], [5], [52] and [53], for example), in our classical explanation of interference a detailed “micro-causal” account of the corresponding kinematics can be given: Firstly, we note that the last term in Eq. (3.7), which is responsible for the genuinely “quantum” behaviour, determines the movement towards the symmetry line. This term is characterized by the product of the vector $\bar{\mathbf{u}}_1 - \bar{\mathbf{u}}_2$ and $\sin \varphi$, the combined effect of which results in the kinks typical for Bohmian trajectories (Fig. 2.1).

Thus, in the cases where the trajectories come from a relative maximum (bright fringe), the particles loose velocity/momentum in the direction towards the symmetry line and cross over into the area of the adjacent relative minimum (dark fringe). From there, they gain velocity/momentum in the direction towards the symmetry line and thus align with the other trajectories of the next bright fringe. In other words, one obtains areas where part of the current (along a relative maximum) is being removed (“depletion”), or where parts of currents flow together to produce a newly formed bright fringe (“accumulation”), respectively. This is in accordance with our earlier description of quantum interference, where the effects of diffusion wave fields were explicitly described by alternating zones of heat accumulation or depletion, respectively [10]. Towards the central symmetry line, then, one observes heat accumulation from both sides, and due to big momentum kicks from the central accumulation of heat energy, the forward particle velocities’ directions align parallel to the symmetry axis. With the crossing-over of particle trajectories being governed by the last, diffusion-related, term on the right hand side of Eq. (3.7), one finds that for $\varphi = 0$

the resulting diffusive current is zero and thus, as total result of the overall kinematics, no crossing is possible. Further, we note that our results are also in agreement with the recently published experimental results by Kocsis et al. [54]. Here we just comment that, as opposed to the Bohmian interpretation, we give a micro-causal explanation of these results solely on classical grounds.

4. THE MEANING OF THE RELATIVE PHASE: QUANTUM SUPERPOSITION, MODULAR MOMENTUM AND THE NONLOCALITY OF THE PATH EXCITATION FIELD

Although we have obtained the usual quantum mechanical results, we have so far not used the quantum mechanical formalism in any way. However, upon employment of the Madelung transformation for each path j ($j = 1$ or 2),

$$\Psi_j = R e^{iS_j/\hbar}, \quad (4.1)$$

and thus $P_j = R_j^2 = |\Psi_j|^2 = \Psi_j^* \Psi_j$, with the definitions (2.3) and $v_j := \nabla S_j/m$, $\varphi = (S_1 - S_2)/\hbar$, and recalling the usual trigonometric identities such as $\cos \varphi = \frac{1}{2}(e^{i\varphi} + e^{-i\varphi})$, etc., one can rewrite the total average current (3.7) immediately as

$$\begin{aligned} \bar{\mathbf{J}}_{\text{tot}} &= P_{\text{tot}} \bar{\mathbf{v}}_{\text{tot}} \\ &= (\Psi_1 + \Psi_2)^* (\Psi_1 + \Psi_2) \frac{1}{2} \left[\frac{1}{m} \left(-i\hbar \frac{\nabla(\Psi_1 + \Psi_2)}{(\Psi_1 + \Psi_2)} \right) + \frac{1}{m} \left(i\hbar \frac{\nabla(\Psi_1 + \Psi_2)^*}{(\Psi_1 + \Psi_2)^*} \right) \right] \\ &= -\frac{i\hbar}{2m} [\Psi^* \nabla \Psi - \Psi \nabla \Psi^*] = \frac{1}{m} \text{Re} \{ \Psi^* (-i\hbar \nabla) \Psi \}, \end{aligned} \quad (4.2)$$

where $P_{\text{tot}} = |\Psi_1 + \Psi_2|^2 =: |\Psi|^2$. The last two expressions of (4.2) are the exact well-known formulations of the quantum mechanical probability current, here obtained without any quantum mechanics, but just by a re-formulation of (3.7). In fact, it is a simple exercise to insert the wave functions (4.1) into (4.2) to re-obtain (3.7).

It is important to note that while the total wave-function $\Psi = \Psi_1 + \Psi_2$ obviously obeys the quantum mechanical superposition principle, no linear superposition principle holds for our total current $\bar{\mathbf{J}}_{\text{tot}}$, as can easily be seen from Eq. (3.7). In accordance with 't Hooft's arguments [40], we have thus demonstrated with an explicit model how the quantum

mechanical superposition principle is only a calculatory means to describe the effects of sub-quantum processes, which are actually to be understood as complex behaviours of ontological microscopic states. This scenario is reassuring, as the constructed linearity of quantum mechanics appears to be “sandwiched” between the sub-quantum and the classical macro-levels, respectively, i.e., levels where the superposition principle does not hold. So, applying linear superposition to states other than those constructed for practical purposes only would lead to wrong predictions about the behaviours of ontological states.[\[55\]](#)

Now that the identity between the versions of our classically derived expression for the average current and the quantum one is established, we have to confront the claim that it was impossible to reproduce quantum results with a classical wave theory, because the meaning of a quantum phase apparently was very different from the meaning of a classical phase. This claim is detailed particularly clearly in [\[56\]](#), where the authors look for a mechanism to explain how the particle, say, at the right, “knows” what is happening at the left slit (i.e., whether the latter is closed or open, for example). Their explanation of interference from the single-particle perspective is based on non-local Heisenberg equations of motion for “modular variables” like the modular momentum, for example. With p denoting the usual momentum, the modular momentum is defined as $p_{mod} := p \bmod \frac{h}{d} = p - n\frac{h}{d}$, where d is the distance between the slits. As $p \bmod \frac{h}{d}$ has the topology of a circle, nothing changes if in the equations one replaces p with $p - n\frac{h}{d}$. The main argument now is that whereas the ordinary momentum (as well as all its higher moments) is independent of the relative phase between Ψ_1 and Ψ_2 , the modular momentum is not. In other words, the authors claim that the sensitivity of the modular momentum towards changes in the relative phase makes it impossible to apply classical intuitions to double slit interference: As the relative phase is a truly non-local feature of quantum mechanics, they claim, classical notions would have to fail to describe the essence of interference. Physically, therefore, it is a non-local effect of having an open or closed slit to produce a shift in the particle’s modular momentum while the expectation values of its ordinary momentum remain unaffected.

However, as we have shown, we have an identity in the outcomes of our classical calculations with the quantum ones. How can this come about? The answer is clearly given by the fact that the path excitation field is, by its very definition, a non-local field. Now, one may argue that non-local fields should have nothing to do with “classical” physics, but as we have repeatedly stressed, nothing speaks *a priori* against the assumption of some non-locally

distributed zero-point oscillations in a purely classical context. Once this is accepted, the rest follows straightforwardly: instead of using the language of a “rotation in the space of a modular variable”, like the “non-local exchange of modular momentum”, for example, we have discussed and described interference with the non-local path excitation field, where the angle between two unit vectors representing the respective velocity fields originating from the two slits is given by the relative phase. As in [56] the parameter relevant to describe the effects of an open or a closed slit, respectively, is given by the distance between the slits appearing in the modular momentum approach, so does this same distance also appear in our description, i.e., it is explicitly given in the formula for the relative phase φ in Eq. (2.23). Thus, the appearance of the distances X in our expression for φ essentially demonstrates the latter to be a non-local one. For example, the closing of one slit at either $-X$ or $+X$ has an immediate effect in that for $v_x = 0$ the last term of the relative phase (2.23) is changed by a factor of $\frac{1}{2}$.

Moreover, if in Eq. (2.23) one discards the diffusion-related term, then φ becomes “classical” in the usual sense of the word, and it is only then that the quantum results could not be reproduced any more. Similarly, in Eq. (3.7) it is the last term proportional to $\sin \varphi$ which determines the genuinely quantum nature of the whole expression, and it is there where via the nonlocality of the “diffusive” velocities \mathbf{u}_i the nonlocality of quantum mechanics becomes manifest. We have thus shown why our classical approach can produce the results in full accordance with quantum mechanics. A more detailed discussion of nonlocality and entanglement within our scheme will be the subject of a paper in preparation.

5. CONCLUSIONS

We have introduced a quantum as an emergent system by considering “particles” as oscillators (“bouncers”) coupling to regular oscillations of the “vacuum’s” zero-point field, which they also generate. Among other features, the dynamics between the oscillator and the “bath” of its thermal environment can be made responsible not only for Gaussian diffraction at a single slit [11], but also for the well-known interference effects at a double slit [15, 48]. We have also shown how the model entails the existence of a path excitation field, i.e., a field spanned by the average velocity fields $\bar{\mathbf{v}}(\mathbf{x}, t)$ and $\bar{\mathbf{u}}(\mathbf{x}, t)$, respectively, where the latter refer to diffusion processes reflecting also the stochastic parts of the zero-point field. We have

derived, on the basis of classical physics only, the exact intensity distribution at a screen behind a double slit, as well as the details of the more complicated particle current, or of the Bohmian particle trajectories, respectively. In a simple computer simulation, we have modeled quantum interference with simple classical rules employing well-known techniques to model diffusion processes [57].

Moreover, we have refuted claims about the impossibility to model quantum interference with any classical (and thus also our) model. The decisive feature of said claims, apparently without a classical, or “natural”, explanation, is the non-local effect of opening or closing one slit on a particle going through the other slit, an effect which manifests itself in a changed expression for the relative phase. However, we were able to show explicitly within our classical approach that the path excitation field in this case must change to produce exactly the same effect on the relative phase. This both qualifies as a truly non-local effect within our approach and provides the identity with the usual quantum mechanical predictions.

Finally, upon comparison with the usual quantum mechanical formalism, we have demonstrated with our explicit model how the quantum mechanical superposition principle is only a calculatory means using non-ontological wave-functions to describe the effects of sub-quantum processes, which are actually to be understood as complex, nonlinear behaviours of ontological microscopic states.

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